Assessing Extreme Value Analysis to predict rare events from the Global Terrorism Database

Lekha Patel[*] Lyndsay Shand[*] J. Derek Tucker[*] Gabriel Huerta[*]

Abstract
Extreme value methods have commonly been used to predict and quantify uncertainty around environmental or climatological events that could have high impact on human casualties or costs (e.g. earthquakes, hurricanes, flooding, wildfires). In this work, our focus is to study the number of casualties as the variable of interest, from the Global Terrorism Database (GTD) for a particular region and time frame and characterize events via finding extreme observations and fitting both a Generalized Extreme Value (GEV) and Generalized Pareto Distribution (GPD) to this data. We assess whether the goodness of fit of the GEV and GPD parameters are adequate for our framework. For the latter, we also provide graphical representations of predicted 95% and 99% quantiles based on our models and compare these to the actual data. The results of these analyses are a building block into the development of a representative Bayesian hierarchical model that fully characterizes the spatial-temporal relationships present in extreme events from the GTD.

Key Words: Extreme value analysis, Global Terrorism Database, Generalized Pareto distribution, Generalized Extreme Value

1. Introduction

Threats to our national security can be reduced by the detection, prevention and deterrence of unlikely extreme events. It is well understood that it is not the obvious likely events that cause distress, but the rarest unanticipated events such as a new terrorist group arising where no one is looking. Predicting the occurrence of a rare event that has never been seen before is an extremely challenging problem, yet to be solved. A feasible approach to anticipating the unlikely is to quantify the probability of an extreme event and the related uncertainty or risk for long term monitoring and integration into decision making. This approach aims to reduce the severity of the "surprise" events and would allow time for decision makers to have options available when needed.

In this paper, our aim is to assess the viability of extreme value analysis to extreme terrorism events and discuss potential research avenue for their statistical quantification. While traditional statistical models seek to describe the overall distribution of such events, extreme value analysis is concerned with characterizing events that lie in the tails or margins of these distributions. Extreme value methods have commonly been used to predict and quantify uncertainty around environmental or climatological events that could have high impact on human casualties or costs (e.g., earthquakes, hurricanes, flooding, wildfires), see for example, Coles (2001). From a statistical point of view, extreme value data can be modeled with two types of approaches. The first, relies on calculating a sequence of maximum (or minimum) values over blocks of data, e.g. monthly or yearly maxima (minima) and then fitting it to its large sample distribution, the Generalized extreme value (GEV) distribution. The second, finds observations that exceed a given threshold (or are

[*]Statistical Sciences, Sandia National Laboratories, PO Box 5800 MS 1202, Albuquerque, NM 87185

below a threshold) and fits the well approximated Generalized Pareto Distribution (GPD) to these exceedance values. The latter method is also known as a Peaks over Threshold (PoT) approach and can be a more efficient use of the raw data rather than only modeling the block maxima. The utilization of both methods are application specific but have not been considered extensively to assess risks of extreme events with national security implications. In this work, we aim to utilize extreme value theory to this application in a spatio-temporal manner by assessing empirical goodness of fit. This model is therefore intended to quantify and predict a future extreme national security event in space and time and characterize the rarity of the event.

1.1 Global Terrorism Database

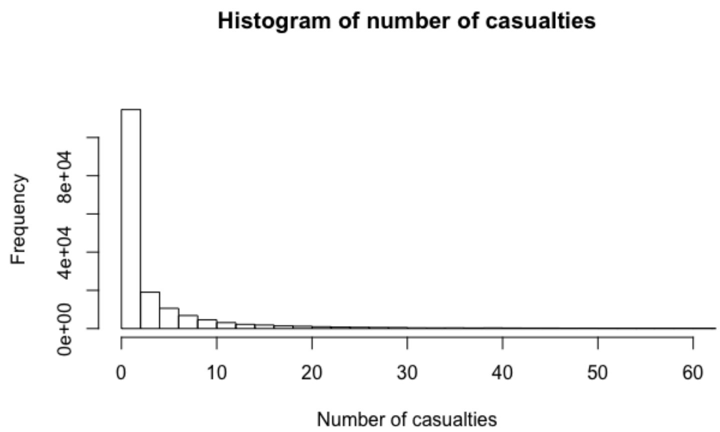
To motivate the proposed framework, we focus our work on the Global Terrorism Database (GTD)[1]. The GTD includes information on more than 190,000 terrorist attacks from 199 different countries occurring from 1970 through 2018. Along with the attack time (day, month and year) and location (latitude and longitude) of each recorded terrorism event, detail is also given on the attack type, weapon(s) used, nature of the target, casualties, injuries, and the group responsible (when available). While extreme value theory has been shown to be a valid approach for predicting the risk of catastrophic terrorism events in Mohtadi and Murshid (2009) using a more limited data set than the GTD, analysis has not been extended to appropriately incorporate spatial or other critical descriptive information of the terrorism events. In order to be able to develop a modern framework of extreme value analysis that integrates the temporal and spatial effects of GTD variables through an appropriate statistical model, it is important to first analyze the viability of extreme valued distributions to extreme events described in this database.

In order to model global terrorism attacks using either the GEV or GPD distributions, we first require an extreme event to be defined. Depending on the question of interest, an extreme attack could either be characterized as occurring in an unusual or unexpected space-time location, or as an extremely rare type such as an event perpetrated by a new terrorist group or an event reporting a high magnitude of fatalities or causalities. The former of the two requires a further definition of what constitutes as an "unexpected" space-time location and could therefore be difficult to formally formulate in comparison to the latter. We therefore turn to rare type modeling and begin, in a similar manner to Mohtadi and Murshid (2009), by defining an extreme attack as one that results in a high reporting of casualties. The number of casualties here is calculated as the number of deaths and injuries resulting from an attack.

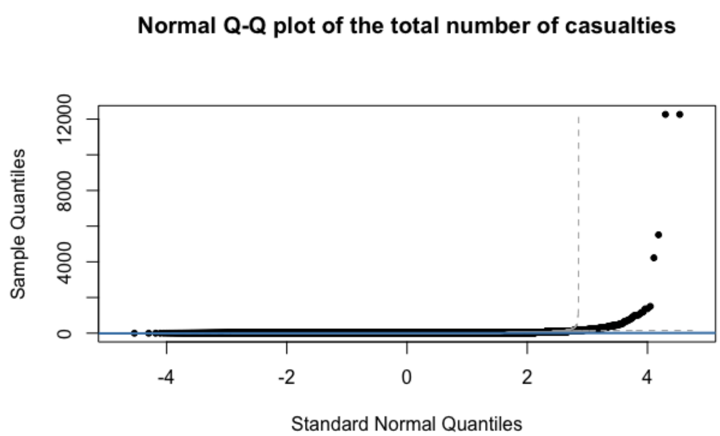
Figure 1a shows a histogram of the total number of casualties reported from the GTD (across all space-time events), with its respective Q-Q plot (Figure 1b) indicating a zero-inflated heavy tailed distribution.

On a country specific level, the countries which have quoted the most deadly attacks in the GTD are: Iraq, Pakistan, Afghanistan, India and Colombia (see Figure 2). Calculated empirical CDFs (Figure 3a) and quantile plots (Figure 3b) suggest that the distributions between pairs of the most deadly countries (Iraq/Afghanistan, India/Pakistan/Colombia) are similar and heavy tailed. These exploratory plots indicate first that extreme valued distributions seem applicable (using our definition of an extreme attack) for this dataset, and second, that there appears to be

[1] This is an open-source database which can be downloaded at https://www.start.umd.edu/gtd/.



(a) Histogram of all casualties.



(b) QQ-plot of all casualties.

Figure 1: Plots of all global casualties during 1970-2018.

a spatio-temporal contribution on the underlying distribution of extreme events in any one country. This effect may be heightened by specific spatio(temporal) regions in specific countries, enabling site specific attack regions to be either identified or incorporated in any corresponding analysis.

1.1.1 Data challenges

Although the GTD contains numerous accounts of attacks, the quality of this data may affect subsequent extreme value analysis of extreme events. For example, there are many missing entries on one or more of the explanatory covariates, number of casualties and the time stamps of events. Furthermore, many of spatial locations of events are also non-exact, with many entries' exact locations either incorrect or being approximated to the longitudes and latitudes of their nearest cities. Furthermore, it is also evident that many countries, especially those with attacks from before 2000, have underreported the data, in terms of intensity and number of reported causalities. This would cause obvious problems with analysis conducted

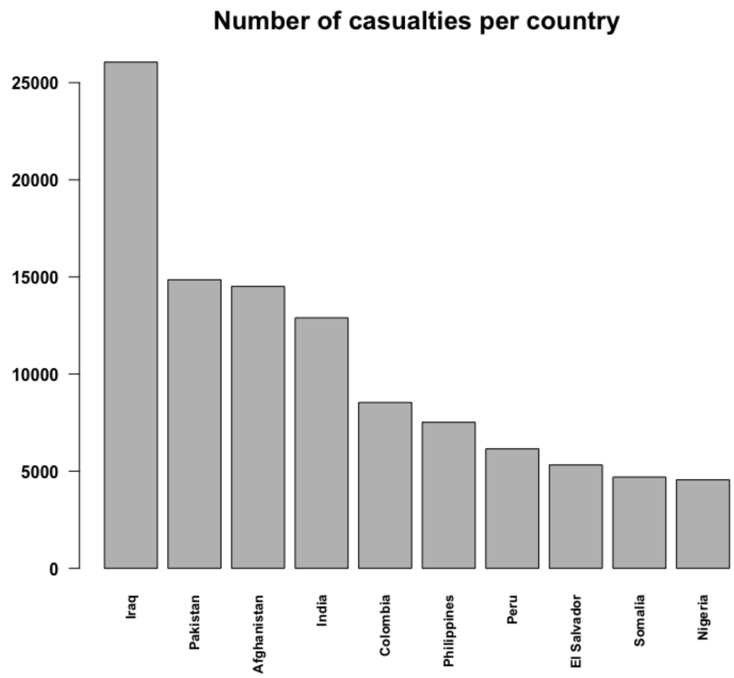
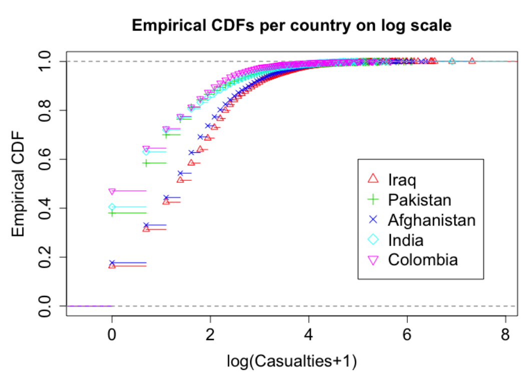
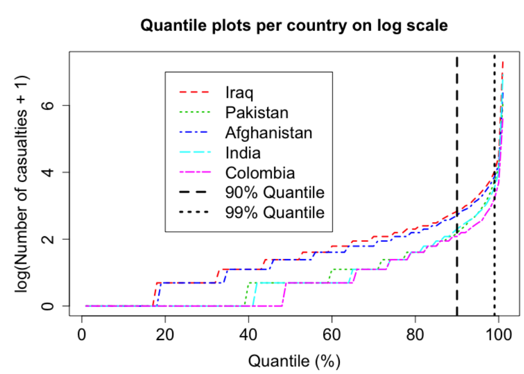


Figure 2: Histogram of causalities per country.



(a) Empirical CDF on log scale.



(b) Empirical quantiles on log scale.

Figure 3: CDF and quantile plots of casualties in the five most deadly countries during 1970-2018.

during these time periods. Although the number of attacks reported for the most deadly countries (see Figure 2) is seemingly large enough for statistical inference, data limitations make EVA difficult for other countries e.g. the USA and UK from which there is not enough data to pursue significant analysis. This therefore limits our focus to country (or site) specific regions in the world where resulting analyses can provide meaningful interpretations.

1.1.2 Structure

The outline of this paper is as follows. In Section 2, we mathematically formulate the Generalized Extreme Value (GEV) and Generalized Pareto distribution (GPD) appropriate for conducting extreme value analysis. In Section 3, we assess goodness of fit of these distributions applied to spatio-temporal casualty data resulting from extreme attacks analyzed in the five more historically attacked countries, as given by the GTD. Finally, in Section 4 we conclude with a discussion on future research directions that incorporate extreme value analysis for this dataset.

2. Preliminaries

In this section, we describe the relevant mathematical background that form the basis of typical extreme value analyses. Specifically, the Generalized Extreme Value (GEV) distribution is formulated for block maxima models and the Generalized Pareto distribution (GPD) for threshold based models.

2.1 Block maxima models

Letting $X_1, X_2, \ldots, X_n$ be a sequence of independent random variables having a common distribution function F, block maxima models focus on the statistical modeling of

$$M_n = \max(X_1, X_2, \ldots, X_n),$$

and a block is considered a period e.g. monthly or annually that each contain observations of $X_1, \ldots, X_n$.

In practice, the distribution F is unknown. To circumvent this, the Fisher-Tippett-Gnedenko theorem (Coles, 2001) states that if there exist a sequence of constants (a_n, b_n) with $a_n > 0$, then the limit

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{M_n - b_n}{a_n} \leq z\right) = G(z).$$

Here, if G is non-degenerate then G belongs to one of the Gumbel, Fréchet or Weibull distribution families. These three families can be succinctly grouped into the family of the Generalized Extreme Value (GEV) distribution. Specifically, if $Z \sim \text{GEV}(\mu, \sigma, \xi)$, then its CDF takes the form

$$G(z) := \mathbb{P}(Z \leq z) = \exp\left(-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right), \tag{1}$$

defined on $\{z : 1 + \xi(z-\mu)/\sigma > 0\}$. Here, $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ is the scale parameter $\xi \in \mathbb{R}$ is the shape parameter. Furthermore, renormalization implies that

$$\begin{aligned}\mathbb{P}(M_n \leq z) &\approx G\left(\frac{z-b_n}{a_n}\right)\\ &= G^*(z),\end{aligned}$$

which is another member of the GEV family (Coles, 2001).

This enables the distribution of block maxima (or minima) to be well approximated by distributions from the GEV family, from which maximum likelihood estimation is in general numerically straight-forward to recover the unknown parameters from given data.

2.2 Threshold models

While distributions from GEV family are subject to favorable asymptotic properties (Coles, 2001), one of the main drawbacks from utilizing the GEV in this manner is that the data in blocks $X_1, X_2, \ldots, X_n$ is often wasted via computation of their maximum (or minimum), frequently resulting in insufficient data for the GEV to be an accurate modeling tool. An alternative approach, which endeavors to utilize data that are observed above a particular threshold, are threshold based models.

Letting $X_1, X_2, \ldots, X_n$ again be a sequence of independent random variables having a marginal common distribution function F, threshold models define an extreme event X from this sequence as that which exceeds some high threshold u. Specifically, its conditional probability is given as

$$\mathbb{P}(X > u + x | X > u) = \frac{1 - F(u+x)}{1 - F(u)} \quad x > 0.$$

Since in practical applications the distribution F is commonly unknown, an approximation to $\mathbb{P}(X > u + x | X > u)$ is necessary. Coles (2001) uses the GEV distribution defined in (1) for large n and u, to yield

$$\mathbb{P}(X \leq u + x | X > u) = 1 - \left(1 + \frac{\xi x}{\sigma + \xi(u-x)}\right)^{-1/\xi}, \tag{2}$$

defined on $\left\{x : x > 0, \left(1 + \frac{\xi x}{\sigma + \xi(u-x)}\right) > 0\right\}$.

Re-parameterizing (2), the CDF of $X \sim \text{GPD}(u, \sigma, \xi)$ is given by

$$\mathbb{P}(X \leq x) = \begin{cases} 1 - \left(1 + \frac{\xi(x-u)}{\sigma}\right)^{-1/\xi} & \text{for } \xi \neq 0 \\ 1 - \exp\left(-\frac{x-u}{\sigma}\right) & \text{for } \xi = 0, \end{cases} \tag{3}$$

where the support of X satisfies $x \geq u$ when $\xi \geq 0$ and $u \leq x \leq u - \frac{\sigma}{\xi}$ when $\xi < 0$. Similarly to the GEV, $\sigma > 0$ is the scale parameter, $\xi \in \mathbb{R}$ is the shape parameter and $u \in \mathbb{R}$ is the threshold or location parameter.

This enables the distribution of data over a pre-specified threshold or exceedances to be well approximated by the GPD, from which maximum likelihood estimation is in general numerically straight-forward to recover the unknown parameters from given data. Analyzing threshold exceedances in this manner are commonly referred to as peaks over threshold (PoT) models. Although these methods typically utilize the data more efficiently than block maxima models, selecting a suitable threshold for which the GPD remains a suitable model, without losing efficiency of the data, is not straightforward and remains an active area of research in this field.

The interested reader is directed to Coles (2001) for further information of the GEV distribution, GPD and other extreme valued type constructions.

Country	Time period
Iraq	2004-2018
Pakistan	2002-2018
Afghanistan	2003-2018
India	2002-2018
Colombia	1977-1995

Table 1

3. Extreme value analysis

In order to analyze the viability of extreme value analysis across all space-time terrorism attacks, it is important to assess its performance first at the country level, i.e. in large regions of space where attacks can be viewed independently of other regions and where temporal structures can be emphasized. In this section, we look at the five most (historically) attacked countries, as described in Section 1, within specific time periods that do not suffer too greatly from the data limitations highlighted in Section 1.1.1. The countries analyzed in this section are summarized in Table 1.

In Section 3.1, we use the block maxima approach and fit a GEV distribution to monthly maxima of casualties, both spatially and temporally. In Section 3.2, we use the peaks over threshold approach and fit a GPD distribution to threshold exceedances, including a discussion on threshold selection methods.

3.1 Analysis of block maxima

In order to use the GEV distribution to GTD country level data, we computed the monthly maxima of casualties over the time periods considered. A monthly block for this dataset was favored over a daily block, which would have resulted in too many null observations, and a yearly block which would have resulted in too few data points. Using these datasets, the standard GEV distribution can be fitted using maximum likelihood estimation, and goodness of fit plots (specifically probability, quantile, density and return levels) that use the maximum likelihood estimated parameters, can be assessed.

Figures 4-7 show the monthly maxima with the corresponding GEV goodness of fit plots for Iraq, Pakistan, Afghanistan, India and Colombia. All countries seem to suggest a good distributive fit with the (standard) GEV, although Pakistan and Colombia yield the poorest of quantile fits.

Spatially, attack data can be analyzed for its clustering or inhibitive properties. Estimates of Ripley's K and pair correlation functions are provided in Figures 9a and 9b for each country previously studied and highlight the high spatial clustering of attacks in comparison to a homogeneous (uniform) spatial Poisson process. Furthermore, the GEV data within the time period 2002-2018 in the Middle East region is plotted in Figure 9c, which further highlights attack clusters around cities and regions of known political conflict.

3.2 Analysis of peaks over threshold

In order to use the GPD distribution to GTD country level data, we first observe the casualties that exceed the 99%, 95% and 90% empirical quantiles over the time periods considered. We then use mean residual life and parameter stability plots to determine a suitable threshold for each country. Using these datasets, the standard GPD distribution can be fitted using maximum likelihood estimation, and goodness of fit plots (specifically probability, quantile, density and return levels) that use the maximum likelihood estimated parameters, can be assessed.

Figure 10 shows the exceedances of the data at three empirical quantiles. It is evident that, even in the case of the 99% quantile, the data seems to extremely heavy tailed (with many points lying above this quantile), providing first evidence of the suitability of PoT models.

In order to continue with the GPD fitting, it is necessary to define the threshold u for each country, for which an attack is considered extreme. Threshold selection is indispensable when using the GPD, since choosing a threshold which is too low may invalidate the Generalized Pareto distributive assumption on the tails of the observed data, while picking a threshold which is too large is likely to encounter maximum likelihood convergence issues due to too few points exceeding it. Although there are many methods (Bader et al., 2018) to aid us in doing so, we follow Coles (2001) and use graphical tools. The three main plots we can consider are the mean residual life plot, and the GPD scale and shape stability plots.

The mean residual life (MRL) plot utilizes the fact that for $X \sim \text{GPD}(u_0, \sigma_{u_0}, \xi)$, $\mathbb{E}(X - u | X > u) = \frac{\sigma_{u_0} + \xi u}{1 - \xi}$, for all $u > u_0$, $\xi < 1$. This expectation is a linear function of u. This is important since the unbiased estimate of $\mathbb{E}(X - u | X > u)$, the mean of all excesses above u, are therefore expected to change linearly in u at levels of u for which the GPD is appropriate. This is exactly what the MRL computes over different values of u.

On the other hand, the scale and shape stability plots use the fact that if the GPD is valid at threshold u_0, then excesses of a higher threshold $u > u_0$ also follow a GPD with identical shape parameter ξ and scale parameter $\sigma_u = \sigma_{u_0} + \xi(u - u_0)$. This implies that the modified scale parameter $\sigma^* = \sigma_u - \xi u$ is constant with respect to u. The shape and scale stability plots therefore provide maximum likelihood estimates for the shape and modified scale parameters across different values of u, with a suitable threshold u_0 being that which the parameter estimates are approximately constant above.

A threshold can then be chosen once the MRL becomes approximately linear and/or the shape/scale stability plots become constant. As stated above, it is important to understand the bias/variance trade-off between picking a large enough threshold to render the GPD appropriate, while also having sufficient data above it for model fitting and selection.

Figures 11 - 15 show the MRL and shape/scale stability plots for the countries considered. From these plots, it is evident that the GPD in general provides a good fit at low thresholds with most countries, with the exception of Pakistan, from which a poorer fit is observed. In this case, a larger threshold is required to ensure parameter stability.

The thresholds selected for each country using these plots is given in Table 2. The corresponding GPD goodness of fit plots for Iraq, Pakistan, Afghanistan, India and Colombia are shown in Figure 16. All countries seem to suggest a good distributive fit with the (standard) GPD.

Similarly to the GEV data, estimates of Ripley's K and pair correlation functions are provided in Figures 17a and 17b for each country using GPD data and analogously highlight the high spatial clustering of attacks. The data within the same time period in the Middle East region shown in Figure 17c, however, highlights

Country	Threshold
Iraq	100
Pakistan	60
Afghanistan	50
India	20
Colombia	10

Table 2

a much greater amount of data available through a PoT approach.

4. Conclusions and future work

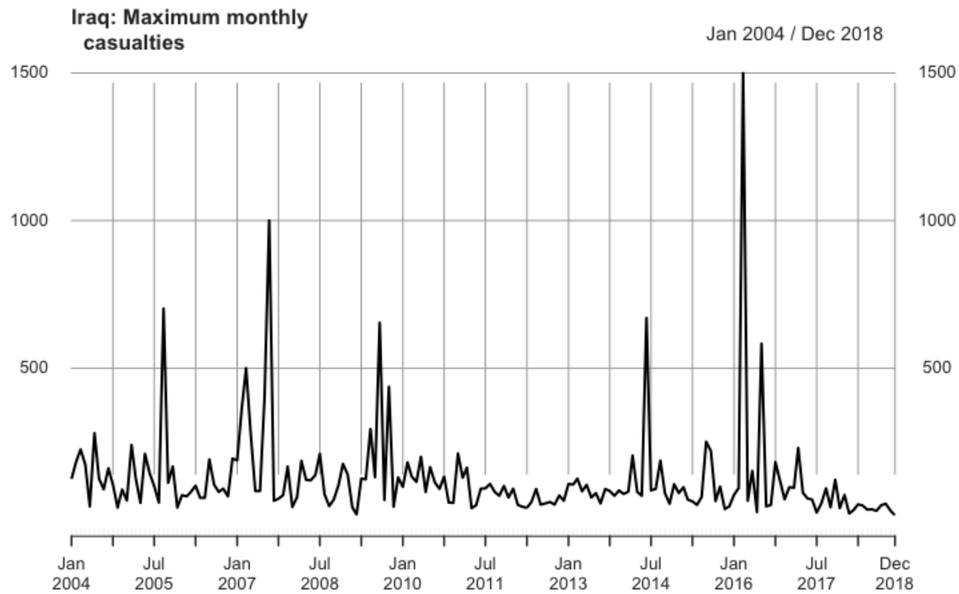
In this paper, we analyzed the effectiveness of using extreme value analysis for the number of casualties produced by global terrorist attacks provided by the Global Terrorism Database (GTD). We first described the GTD and some of the challenges that arise from the available data. In order to apply extreme value analysis to casualty data, we highlighted through empirical quantile and CDF plots, heavy tailed properties for the five most attacked countries in the GTD: Iraq, Pakistan, Afghanistan, India and Colombia. These countries then formed the basis of extreme value analysis through block maxima and points over threshold (PoT) approaches. The former was analyzed via the Generalized Extreme Value (GEV) distribution over monthly blocks, from which attacks producing the highest number of casualties over pre-specified time intervals bespoke to each country formed the data to evaluate. The latter utilized the Generalized Pareto Distribution (GPD) above a high threshold, which was computed via both high empirical quantile or threshold selection methods for each country studied. Both the GEV and GPD showed high spatial clustering of observed extreme attacks around cities or regions with known political conflict at that time, however, also highlighted differences in the degree of spatial clustering between the countries of interest.

While both block maxima and PoT methods provide compelling evidence that modeling of the GTD can be done effectively using extreme value analysis, the data accretion method resulted in the PoT being utilized with a greater amount of data albeit with the added requirement of (pre-)determining the threshold to conduct the analyses. Nevertheless, in terms of analyzing the risk associated with an extreme attack occurring within a fine spatial-temporal resolution, the GPD would perhaps provide more flexibility in understanding this behavior. With the spatio-temporal analysis provided between countries, it is also evident that a more flexible approach is required to fully recognize the differences in GEV/GPD parameter estimates.

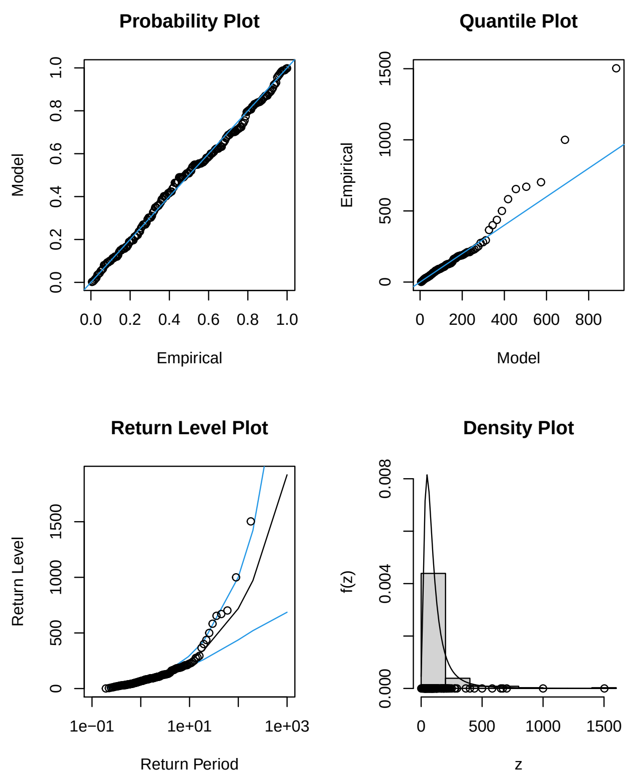
Although hierarchical models for extremes have been extensively studied for extremal data (Gaetan and Grigoletto, 2007; Ghosh and Mallick, 2011; Reich and Shaby, 2012; Yadav, 2019), data challenges and structures may render regression (Casson and Coles, 1999; Chen et al., 2020; Opitz et al., 2018) or copula (Ning, 2017) based approaches difficult. As a specific direction for future research with this dataset and application, a marked spatio-temporal point process approach in a Bayesian hierarchical setting may be more suitable. This method could more flexibly specify the attack distribution in space-time and incorporate the distribution of casualties through the marks, while also addressing the data challenges arising from the GTD. The distribution of marks could be modeled both above and below a threshold (MacDonald et al., 2011), using the analysis presented in this paper, either with a random threshold (Bader et al., 2018) built into the model or fixed at a particular value.

5. Acknowledgments

This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government. This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories, a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525. SAND Number: SAND2020-10848 C.

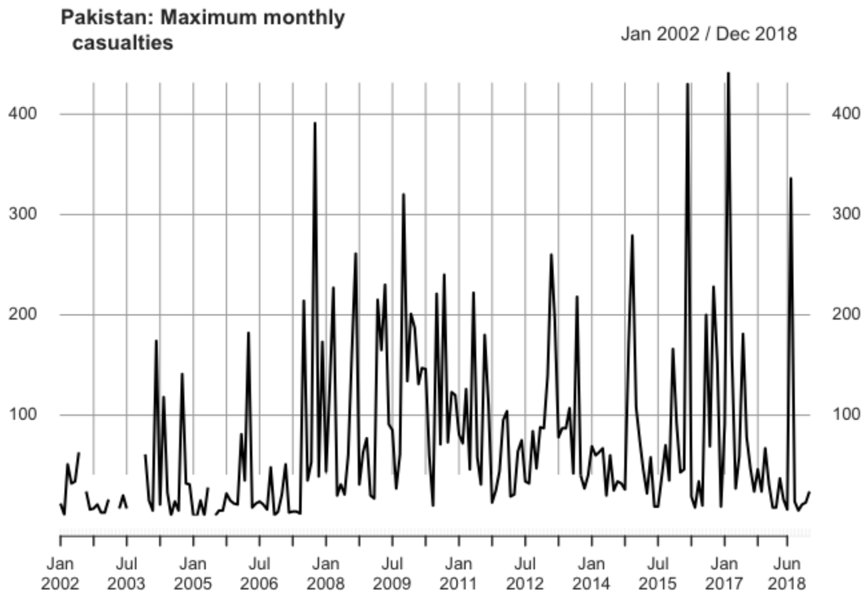


(a) Iraq maximum casualties.

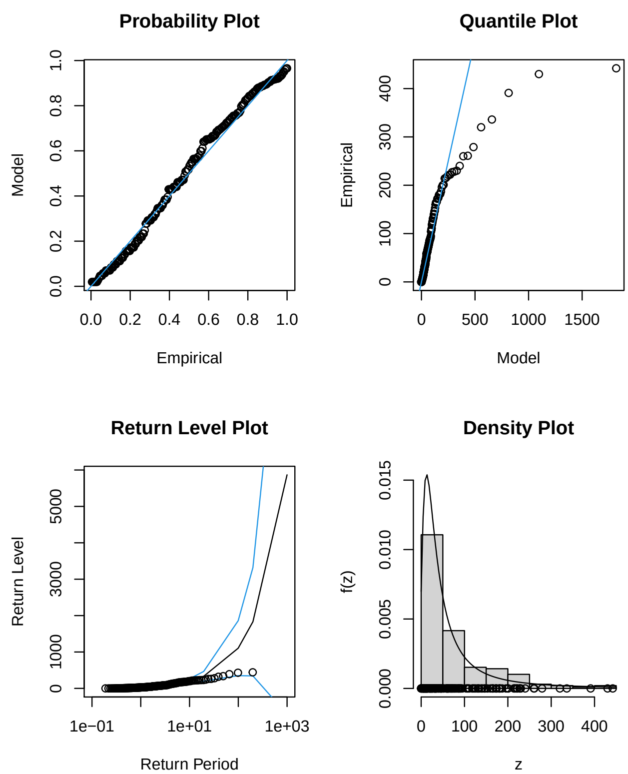


(b) Iraq GEV plots.

Figure 4: Maximum monthly casualties in Iraq and its corresponding GEV goodness of fit plots.

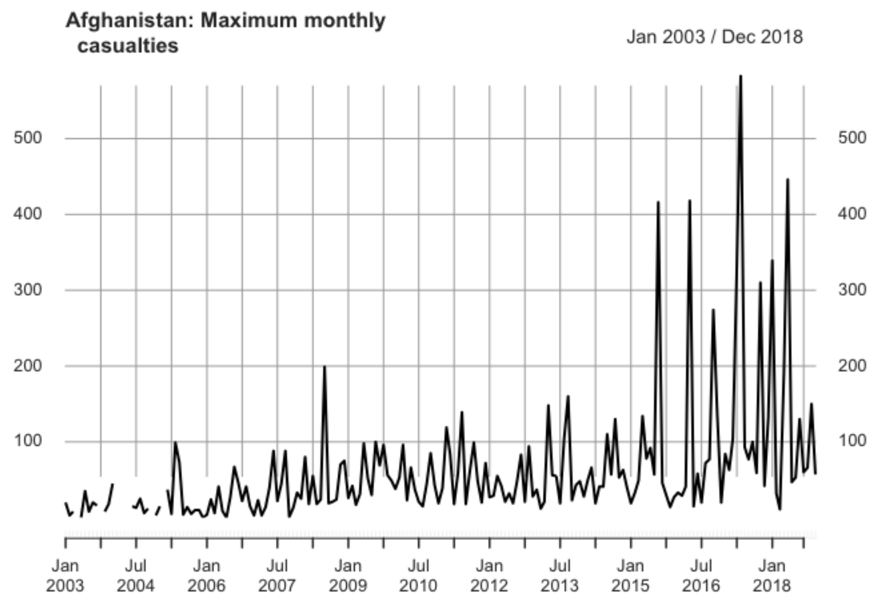


(a) Pakistan maximum casualties.

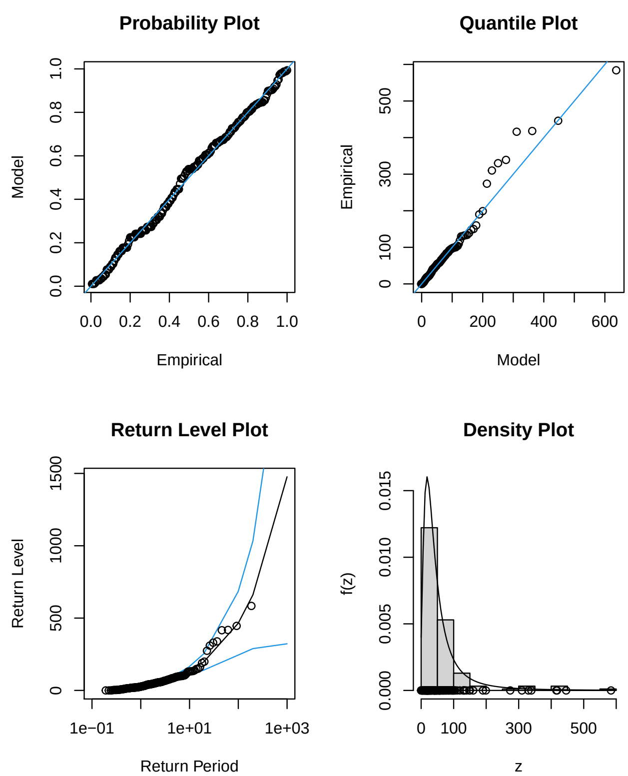


(b) Pakistan GEV plots.

Figure 5: Maximum monthly casualties in Pakistan and its corresponding GEV goodness of fit plots.

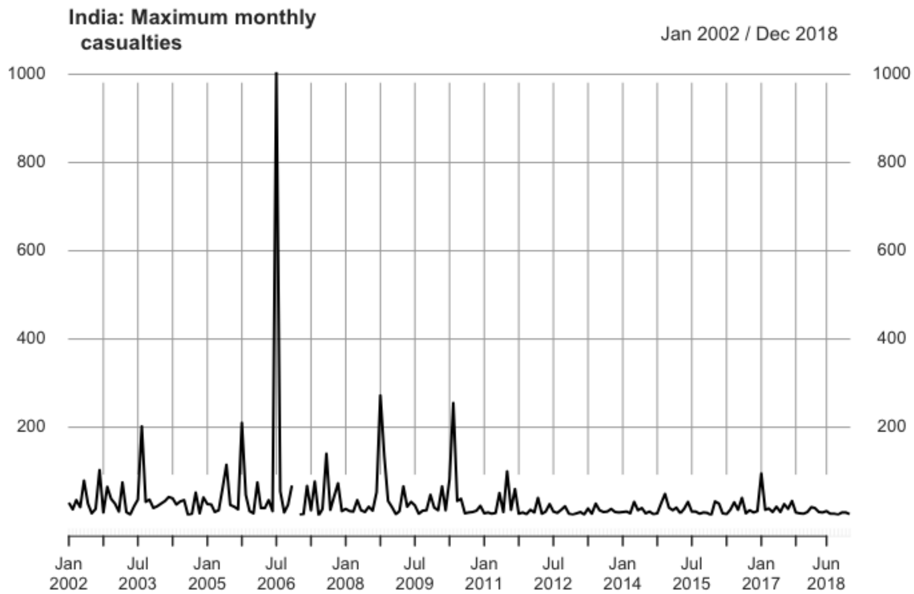


(a) Afghanistan maximum casualties.

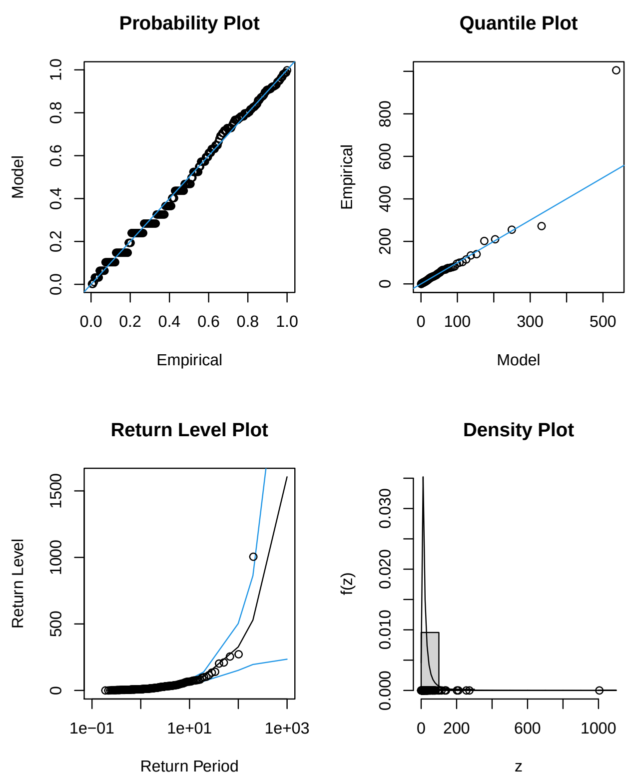


(b) Afghanistan GEV plots.

Figure 6: Maximum monthly casualties in Afghanistan and its corresponding GEV goodness of fit plots.

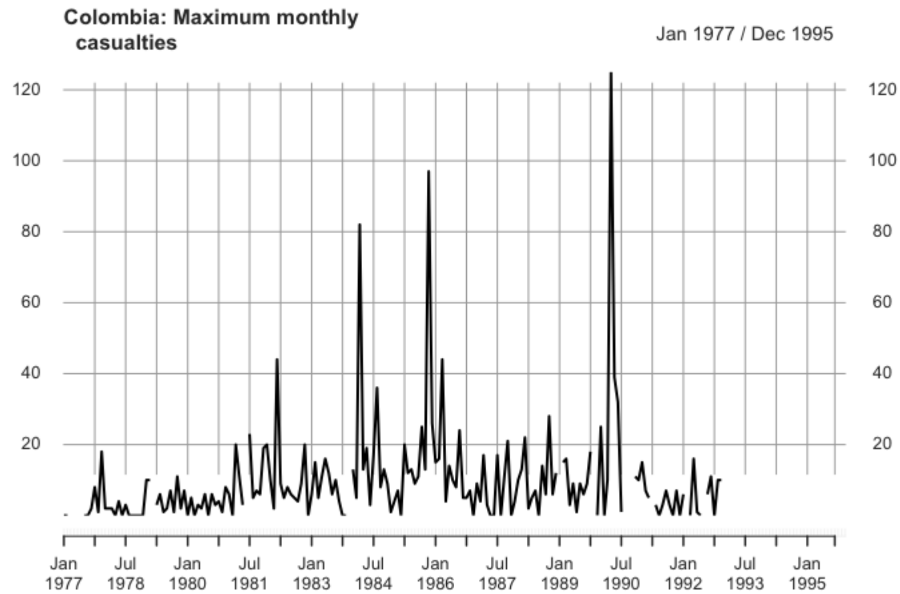


(a) India maximum casualties.

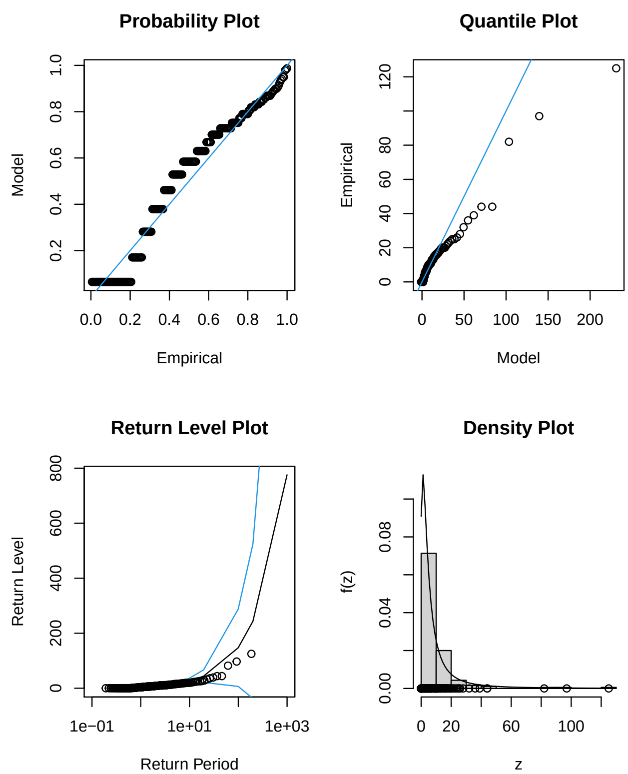


(b) India GEV plots.

Figure 7: Maximum monthly casualties in India and its corresponding GEV goodness of fit plots.

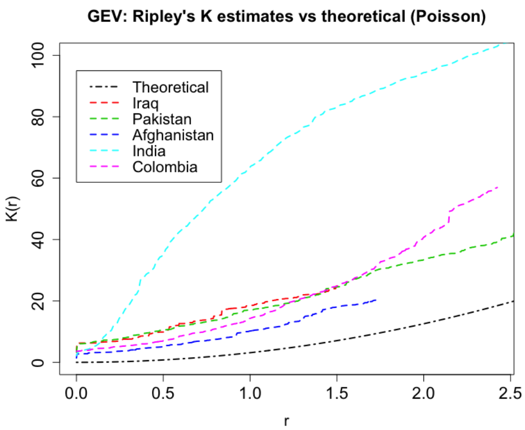


(a) Colombia maximum casualties.

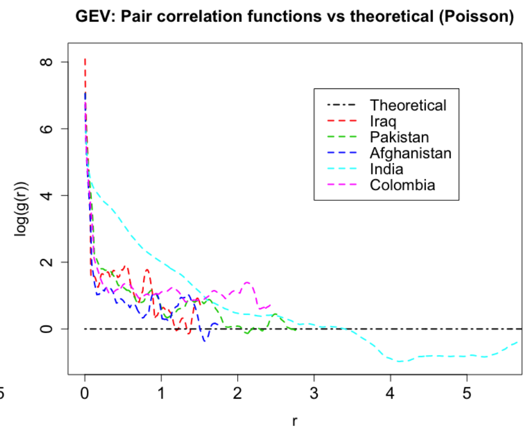


(b) Colombia GEV plots.

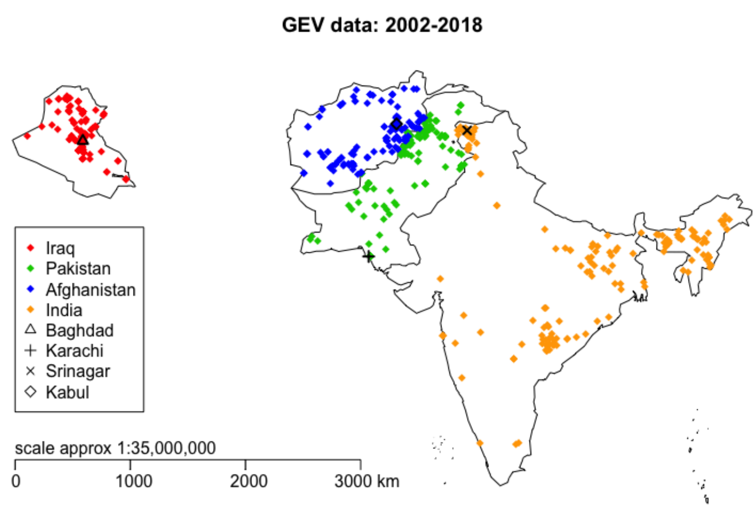
Figure 8: Maximum monthly casualties in Colombia and its corresponding GEV goodness of fit plots.



(a) GEV: Ripley's K function.

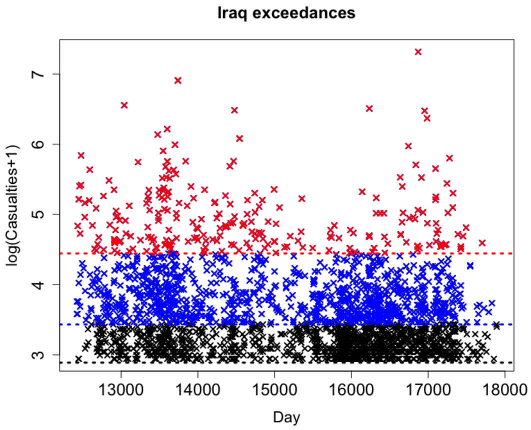


(b) GEV: Pair correlation functions.

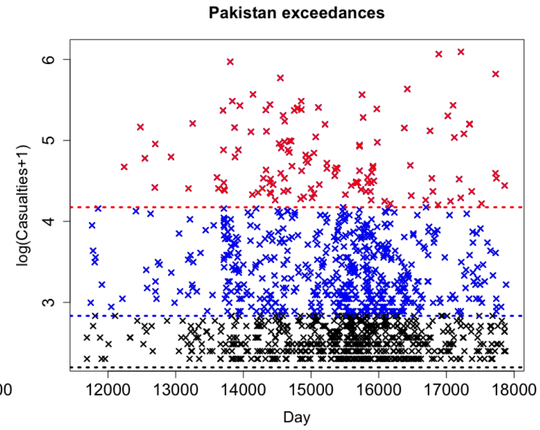


(c) GEV data plotted spatially.

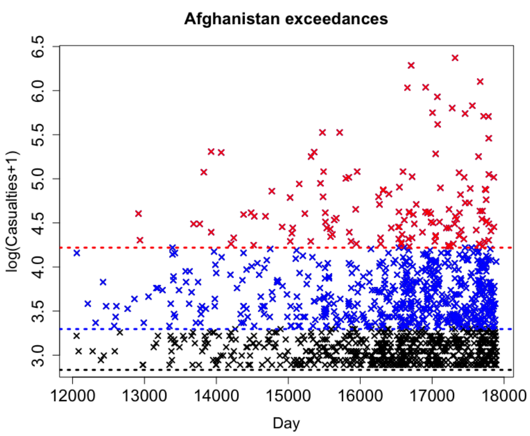
Figure 9: Spatial analyses of block maxima (GEV) attack data.



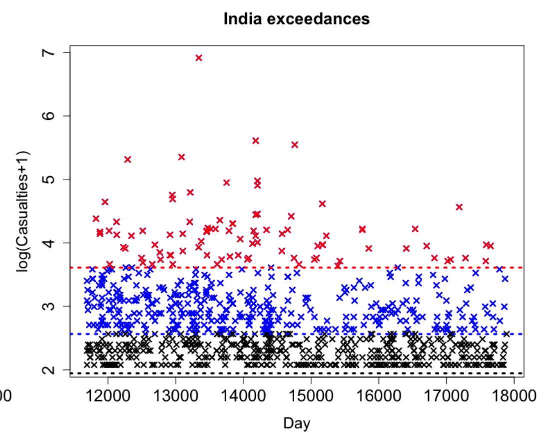
(a) Iraq exceedances.



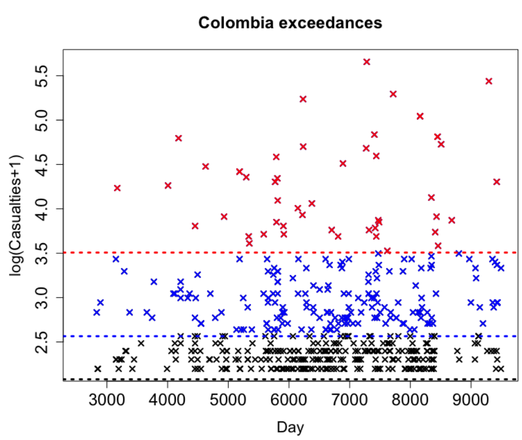
(b) Pakistan exceedances.



(c) Afghanistan exceedances.

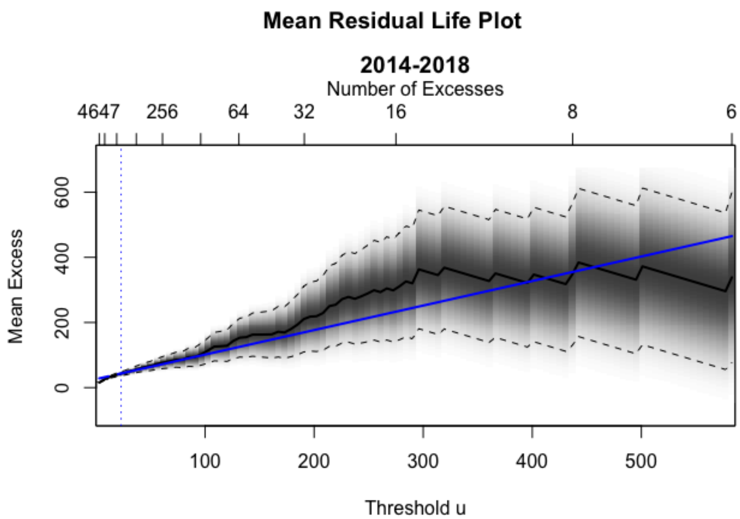


(d) India exceedances.

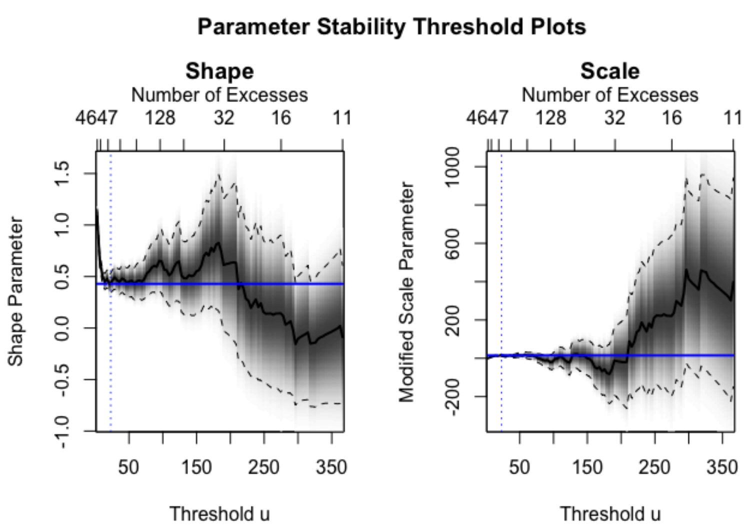


(e) Colombia exceedances.

Figure 10: Observed exceedances (on log scale) at the 90% (black), 95% (blue) and 99% (red) quantiles.

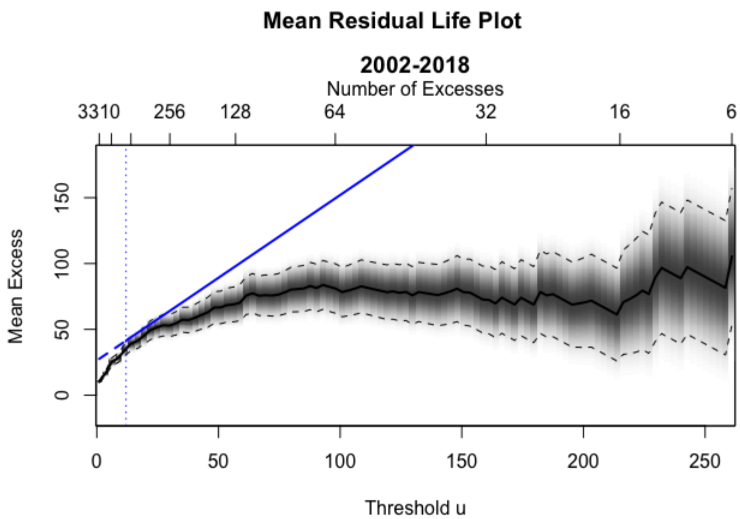


(a) Mean residual life plot.

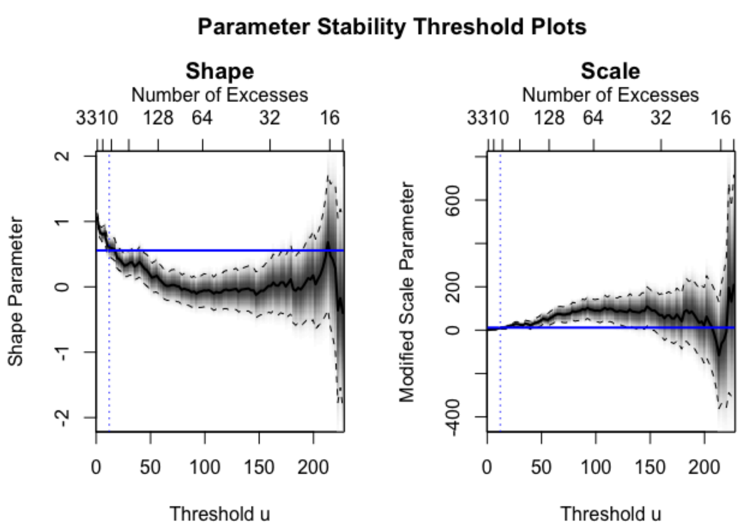


(b) Parameter stability plots.

Figure 11: Iraq: Threshold selection tools.

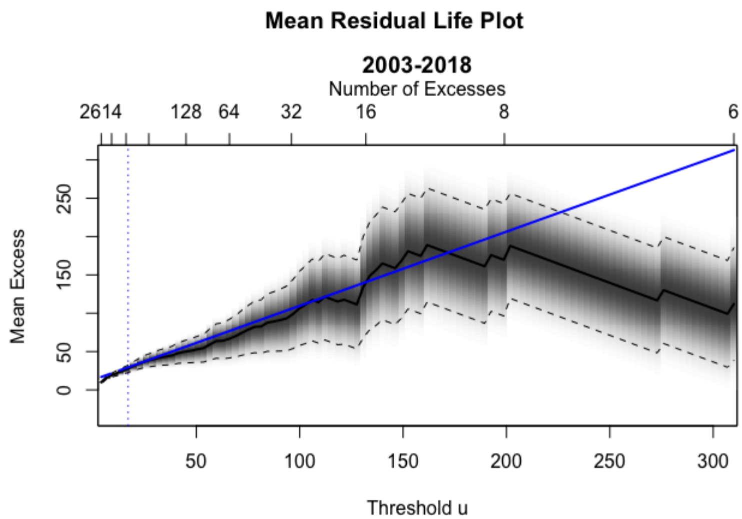


(a) Mean residual life plot.

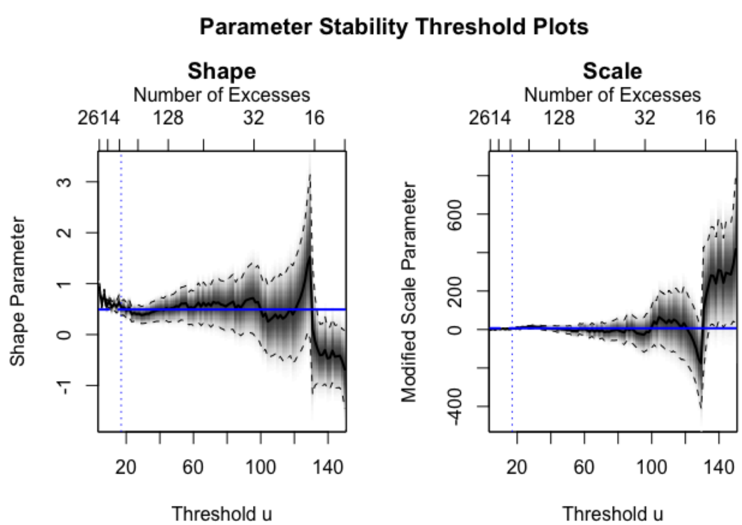


(b) Parameter stability plots.

Figure 12: Pakistan: Threshold selection tools.

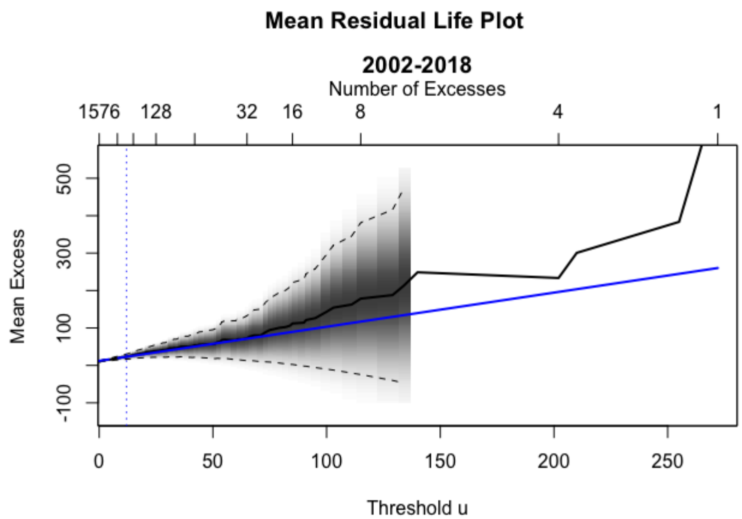


(a) Mean residual life plot.

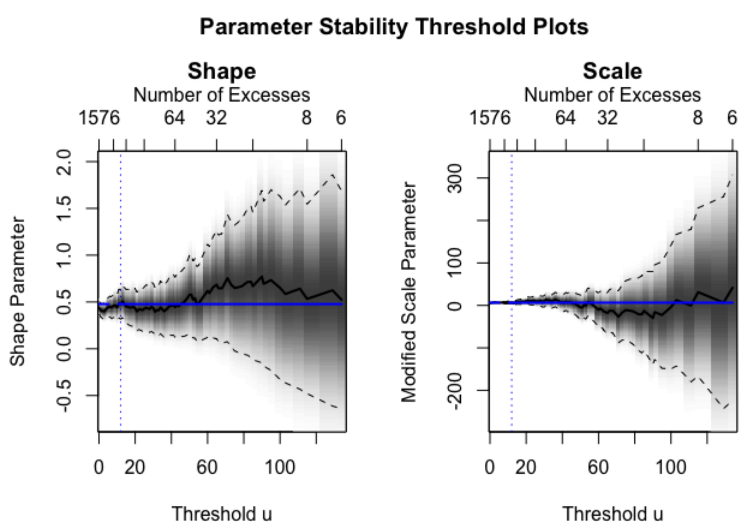


(b) Parameter stability plots.

Figure 13: Afghanistan: Threshold selection tools.

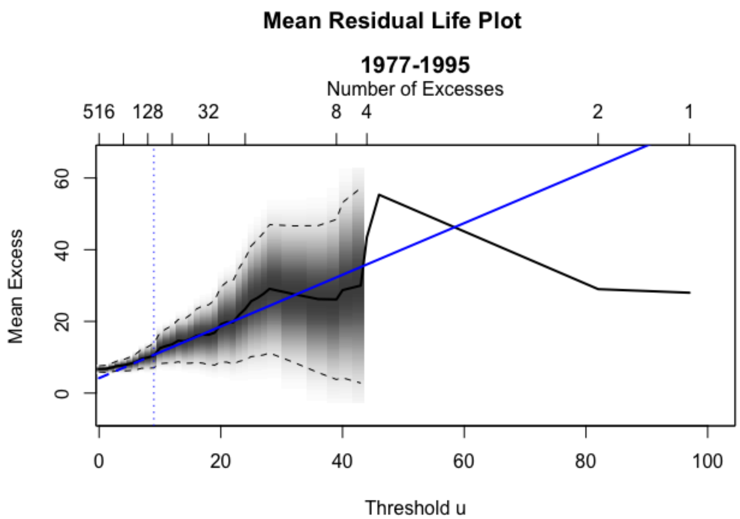


(a) Mean residual life plot.

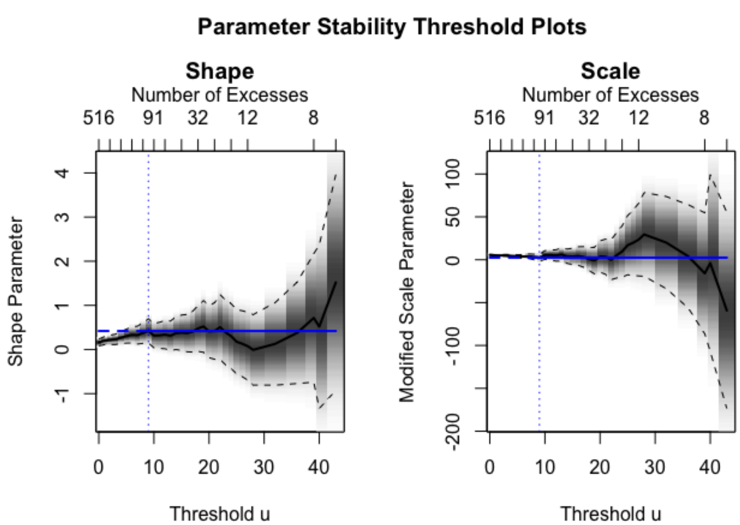


(b) Parameter stability plots.

Figure 14: India: Threshold selection tools.

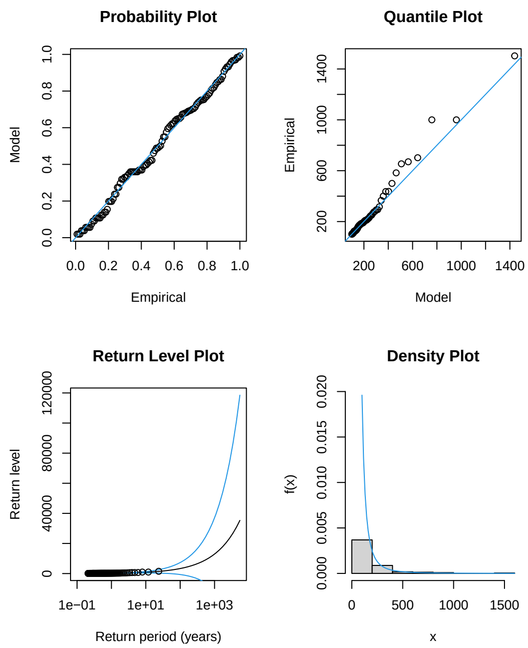


(a) Mean residual life plot.

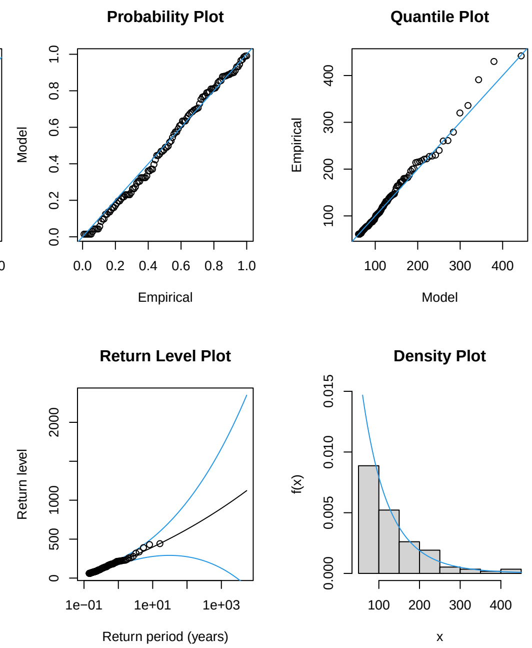


(b) Parameter stability plots.

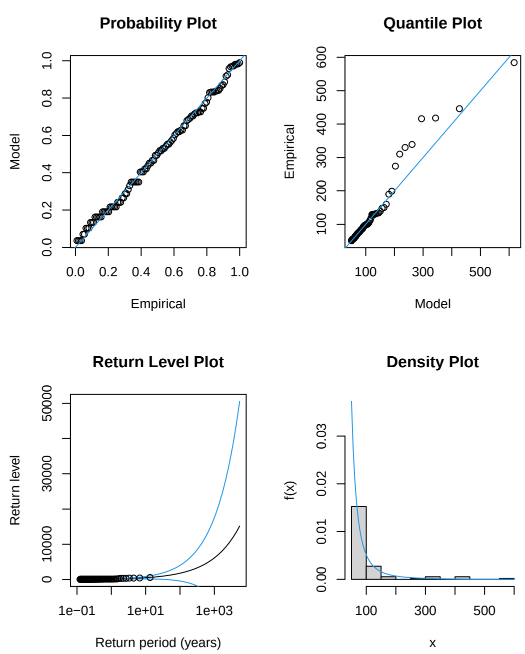
Figure 15: Colombia: Threshold selection tools.



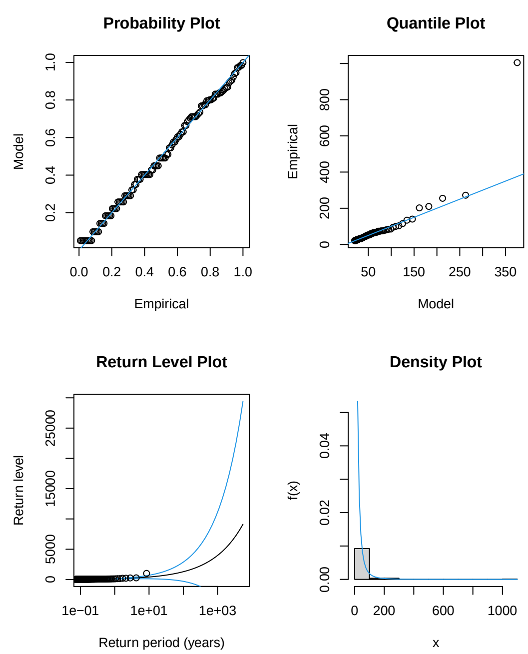
(a) Iraq GPD fit.



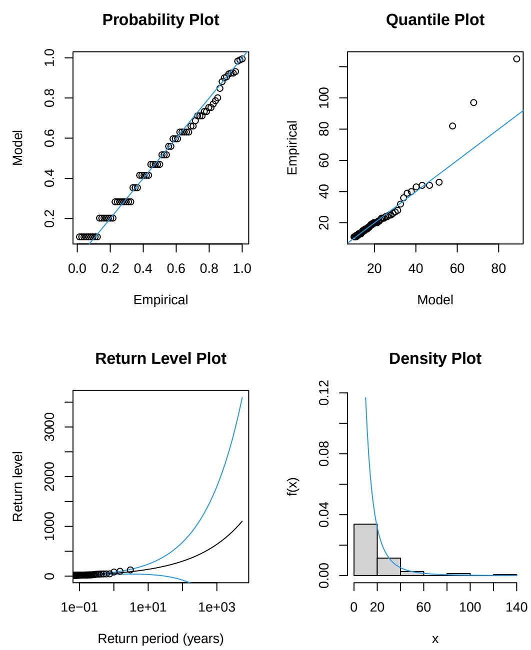
(b) Pakistan GPD fit.



(c) Afghanistan GPD fit.

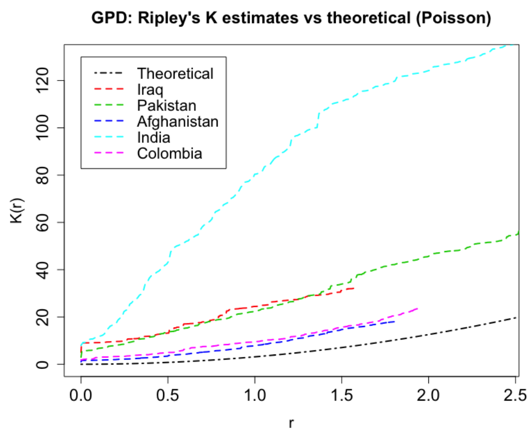


(d) India GPD fit.

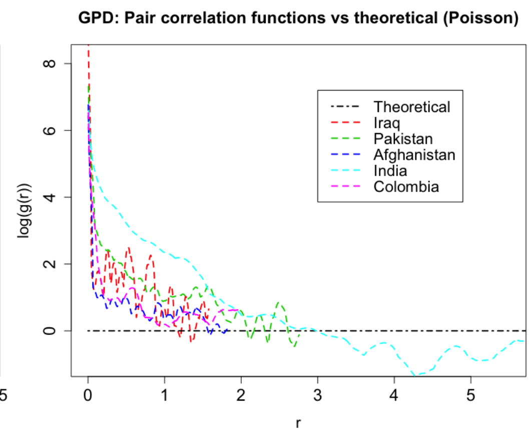


(e) Colombia GPD fit.

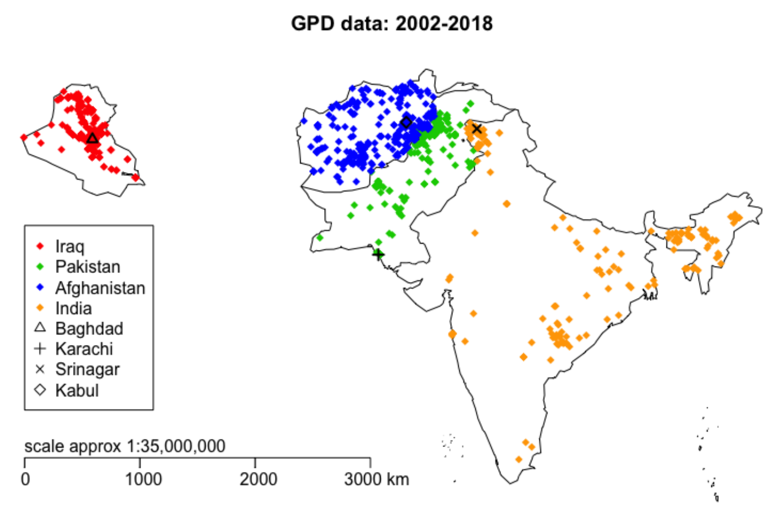
Figure 16: GPD goodness of fit plots.



(a) GPD: Ripley's K function.



(b) GPD: Pair correlation functions.



(c) GPD data plotted spatially.

Figure 17: Spatial analyses of points over threshold (GPD) attack data.

References

Bader, B., J. Yan, and X. Zhang (2018). Automated threshold selection for extreme value analysis via ordered goodness-of-fit tests with adjustment for false discovery rate. Ann. Appl. Stat. 12(1), 310–329.

Casson, E. and S. Coles (1999). Spatial regression models for extremes. Extremes 1(4), 449–468.

Chen, W., S. Castruccio, and M. G. Genton (2020). Assessing the risk of disruption of wind turbine operations in saudi arabia using bayesian spatial extremes. Extremes.

Coles, S. (2001). An introduction to statistical modeling of extreme values. Springer Series in Statistics. Springer-Verlag.

Gaetan, C. and M. Grigoletto (2007). A hierarchical model for the analysis of spatial rainfall extremes. Journal of Agricultural, Biological, and Environmental Statistics 12(4), 434.

Ghosh, S. and B. K. Mallick (2011). A hierarchical bayesian spatio-temporal model for extreme precipitation events. Environmetrics 22(2), 192–204.

MacDonald, A., C. J. Scarrott, D. Lee, B. Darlow, M. Reale, and G. Russell (2011). A flexible extreme value mixture model. Computational Statistics & Data Analysis 55(6), 2137–2157.

Mohtadi, H. and A. P. Murshid (2009). Risk of catastrophic terrorism: an extreme value approach. Journal of Applied Econometrics 24(4), 537–559.

Ning, B; Bloomfield, P. (2017). Bayesian inference for generalized extreme value distribution with gaussian copula dependence. ArXiv.

Opitz, T., R. Huser, H. Bakka, and H. Rue (2018). Inla goes extreme: Bayesian tail regression for the estimation of high spatio-temporal quantiles. Extremes.

Reich, B. J. and B. A. Shaby (2012). A hierarchical max-stable spatial model for extreme precipitation. Ann. Appl. Stat. 6(4), 1430–1451.

Yadav, R; Huser, R. O. T. (2019). Spatial hierarchical modeling of threshold exceedances using rate mixtures. Arxiv.